Name: Claire Date: 9.23

6/10

2.1 Solving for Exponents

$$\boxed{a^n \times a^m = a^{m+n}} \qquad \boxed{a^n \div a^m = \frac{a^n}{a^m} = a^{n-m}} \qquad \boxed{\left(a^n\right)^m = a^{m\times n}}$$

1. Find the missing value in the box for each of the following equations:

a) $3^{\boxed{4}} = 81$ — 4

b) $4^{\boxed{3}} = 64$ — 3

c) $9^{\boxed{3}} = 729$ — 3.

d) $2\times 8^{\boxed{5}} = 65536$ — $8^5 \times 2$.

e) $5^{\boxed{7}} = 78125$ — 7

f) $6^{\boxed{5}} = 7776$ — 5.

2. Simplify each of the following expressions in exponential form:

a) $8\times 2^5 = \boxed{}$ — $2^3 \times 2^5$

b) $\frac{27}{3^2} = \boxed{}$ — $\frac{3^3}{3^2}$

c) $\left(\frac{512}{2^5}\right)^2 = \boxed{}$ — $\left(\frac{2^9}{2^5}\right)^2 = (2^4)^2 = 2^8$

3. Solve for the unknown variable "x" in each of the following expressions:

a) $3^x = 81$

$3^x = 3^4$

$x=4$

b) $2^x = 256$

$2^x = 2^8$

$x=8$.

c) $12^x = 2^6 3^3$

$2^{2x}3^x = 2^6 3^3$

$\begin{cases} 2x = 6 \\ x = 3 \end{cases}$

d) $243 = \frac{3^x}{27}$

$3^5 = \frac{3^x}{3^3}$

$3^8 = 3^x$

$x=8$.

e) $\frac{64}{2^x} = 1024$

$\frac{2^6}{2^x} = 2^{10}$

$2^6 = 2^{10-x}$

$x=4$.

f) $(27)^3 = 3^x$

$3^9 = 3^x$

$x=9$

4. Find the value of variable in each of the following equations:

a) $3^{2x} = 729$

$9^x = 9^3$

$x=3$.

b) $4^{3x} = 128$

$2^{6x} = 2^7$

$x = \frac{7}{6}$

c) $729^x = 81$

$9^{3x} = 9^2$

$x = \frac{2}{3}$

d) $4^{x+1}=65536$

$4^{x+1}=4^8$

$x=7$

e) $25^{2x}=78125$

$5^{4x}=5^7$

$4x=7$

$x=\frac{7}{4}$

f) $36^{x+3}=7776$

$6^{2x+6}=6^5$

$2x+6=5$

$x=-\frac{1}{2}$

5. Find the value of variable in each of the following equations:

$4^x(16)=64$

$4^x=4$

$x=1$

b) $\left(3^{2x}\right)^3=81$

$3^{6x}=3^4$

$6x=4$

$x=\frac{2}{3}$

c) $\frac{16^x}{2^{3x}}=128$

$\frac{2^{4x}}{2^{3x}}=2^7$

$2^x=2^7$

$x=7$

d) $\left(4^{x+1}\right)\left(2^5\right)=65536$

$2^{2x+2+5}=2^{16}$

$2x+7=16$

$x=\frac{9}{2}$

e) $\frac{\left(729^x\right)}{9^{x-3}}=3$

$3^{6x}=3\cdot 3^{2x-6}$

$6x=2x-5$

$4x=-5$

$x=-\frac{5}{4}$

f) $\left(\frac{8^{-1}+2^{-3}}{4^{-3}}\right)^3=32^{-x+1}$

$\left(\frac{2^{-3}+2^{-3}}{2^{-6}}\right)^3=2^{-5x+5}$

$\left(2^4\right)^3=2^{-5x+5}$

$12=-5x+5$

$7=-5x$

$-\frac{7}{5}=x$

6. Given the equation, find the smallest value of x+y:

a. $12^4=2^x3^y$

$2^8\cdot 3^4=2^x3^y$

$x+y=8+4=\boxed{12}$

b) $3969^3=x^69^y$

$\left(7^2\cdot 9^2\right)^3=x^69^y$

$7^6\cdot 9^6=x^69^y$

$x+y=7+6=\boxed{13}$

c) $x^y=y^x \quad (x\neq y)$

$2^4=4^2$

$x+y$

$=2+4$

$=\boxed{6}$

7. Solve for "x" in each of the following equations:

a) $\left(\frac{81^{x+4}}{9^5}\right)^4=\left(\frac{1}{3}\right)^x$

$\left(\frac{3^{4x+16}}{3^{10}}\right)^4=\frac{1}{3^x}$

$3^{16x+24-x}=3^0$

$15x=-24$

$x=-\frac{8}{5}$

b) $\left(\frac{16^{-x}}{32^2}\right)^4=64^{x+1}$

$\left(\frac{2^{-4x}}{2^{10}}\right)^4=2^{6x+6}$

$2^{-16x-40}=2^{6x+6}$

$-16x-40=6x+6$

$-22x=46$

$x=-\frac{23}{11}$

c) $\left(\frac{\left(81^2\right)9^x}{729^3}\right)^{-1}=\frac{\left(3^{3x+1}\right)}{9^x}$

$\frac{3^{18}}{3^{2x+8}}=\frac{3^{3x+1}}{3^{2x}}$

$3^{-2x+10}=3^{x+1}$

$-3x=-9$

$x=3$

8. If $x^2yz^3 = 7^4$ and $xy^2 = 7^5$, then what is the value of xyz? ______ 343

$xy^2 = 7 \cdot 7^4$ $\quad 7^2 \cdot 49 \cdot z^3 = 7^4$

$xy^2 = 7 \cdot 49^2$ $\quad 7^4 \cdot z^3 = 7^4$ $\quad xyz = 7 \times 49 \times 1 = 343$

$x = 7,\ y = 49.$ $\quad z^3 = 1$

$z = 1.$

9. Solve for "x": $2\left(2^{2x}\right) = 4^x + 64$. ______ $x = 3.$

$2^{2x+1} = 2^{2x} + 2^6$

$2^{2x+1} - 2^{2x} = 2^6$

$2^{2x}(2-1) = 2^6$

$2^{2x} = 2^6 \quad x = 3.$

10. Let "a" and "b" be real numbers, with $a > 1$ and $b > 0$. If $ab = a^b$ and $\frac{a}{b} = a^{3b}$, ______ $\begin{cases} a = 4 \\ b = \frac{1}{2}. \end{cases}$

determine the value of 'a'

$ab = a^b$ $\quad \frac{a}{b} = a^{3b}$ $\quad \frac{1}{2}a = a^{\frac{1}{2}}$

$b = a^{b-1} \rightarrow \frac{a}{a^{b-1}} = a^{3b}$ $\quad \frac{1}{2}a = \sqrt{a}.$

$a^{2-b} = a^{3b}.$ $\quad \frac{1}{4}a^2 = a.$

$3b = 2-b.$ $\quad b = \frac{1}{2}.$ $\quad a = 4.$

11. Challenge: If $a = 3^p$, $b = 3^q$, $c = 3^r$, *and* $d = 3^s$ and if *p,q,r, and s* are positive integers, determine the smallest value of $p+q+r+s$ such that: $a^2 + b^3 + c^5 = d^7$ ______ 106.

$p\ 45$ $\quad 3^{2p} + 3^{3q} + 3^{5r} = 3^{7s}$

$q\ 30$ $\quad 45+30+18+13 = 106.$ $\quad 2p = 3q = 5r.$ $\quad 3^{90} + 3^{90} + 3^{90} = 3^{7s}$

$r = 18$

12. If x and y are integers with $(y-1)^{x+y} = 4^3$, then what is the number of possible values for "x"? $\quad 3^{1} \cdot 3^{90} = 3^{7s}$

(A) 8 $\qquad$ (B) 3 $\qquad$ (C) 4 $\qquad$ (D) 5 $\qquad$ (E) 6 $\quad 3^{91} = 3^{7s}.$

(1) $(y-1)^{x+y} = 4^3$ $\quad$ (2) $(y-1)^{x+y} = 2^6.$ $\quad 7s = 91$

$\begin{cases} y-1 = 4 \\ x+y = 3 \end{cases}$ $\quad$ (3) $(y-1)^{x+y} = 64^1$ $\quad s = 13.$

13. Suppose $N = 1 + 11 + 101 + 1001 + \ldots\ldots + 1000\ldots..000001$ (last term has 50 zeroes) When "N" is calculated, and written as a single integer, what is the sum of its digits?

$1 + 11 + 101 + 1001$ $\quad = 1111 \cdots 1162.$

12

113 $\quad 50 \times 1 + 6 + 2 = \boxed{158}$

? 14. If $\cancel{3^{17}} + 3^{17} + 3^{17} + 3^{17} + 3^{17} + 3^{17} + 3^{17} + 3^{17} + 3^{17} + 3^{17} = 81^N$, then the value of ~~N~~ is:

$9 \times 3^{17} = 81^N$

(a) 6 $\qquad$ (b) $\frac{17}{4}$ $\qquad$ (c) $\frac{17}{2}$ $\qquad$ (d) $\frac{19}{4}$ $\qquad$ (e) $\frac{16}{9}$

$4N = 19 \quad N = \frac{19}{4}.$

15. If $x = 2^{12} \times 3^6$ and $y = 2^8 \times 3^8$, what integer z satisfies $x^x y^y = z^z$? (Note: your answer should not be expanded. Instead, write answer as a product of powers of primes.)

$x^x y^y$

$= (2^{12} \times 3^6)^{2^{12} \cdot 3^6} \cdot (2^8 \times 3^8)^{2^8 \times 3^8}$

$= (12^6)^{12^6} \cdot (6^8)^{6^8}$

$= (2^{12})^{2^{12} \cdot 3^6} \cdot (3^6)^{2^{12} \cdot 3^6} \cdot (2^8)^{2^8 \cdot 3^8} (3^8)^{2^8 \cdot 3^8}$

$= 2^{2^{14} 3^7} \cdot 3^{2^{13} \cdot 3^7} \cdot 2^{2^{11} \cdot 3^8} \cdot 3^{2^{11} \cdot 3^8}$

$= 2^{2^{14} 3^7 + 2^{11} 3^8} \cdot 3^{2^{13} \cdot 3^7 + 2^{11} \cdot 3^8}$ $\rightarrow$ Go to the back page.

$= 2^{2^{14}\cdot 3^7 + 2^{11}\cdot 3^8} \cdot 3^{2^{13}\cdot 3^7 + 2^{11}\cdot 3^8}$

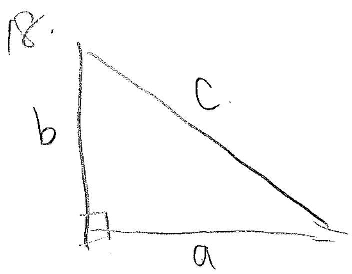
$= 2^{2^{11}\cdot 3^7 (2^3+3)} \cdot 3^{2^{11}\cdot 3^7 (2^2+3)}$

$= (2^{11})^{2^{11}\cdot 3^7} \cdot (3^7)^{2^{11}\cdot 3^7}$

$= (2^{11}\cdot 3^7)^{2^{11}\cdot 3^7}$

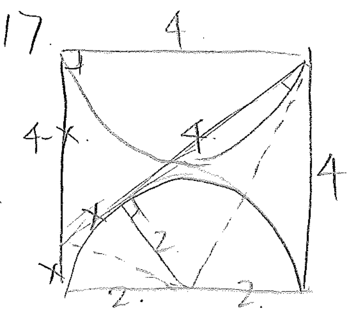
15. $\therefore z = \boxed{2^{11}\times 3^7}$

18.



$\frac{c}{a} = ?$

17.



$\frac{a}{b} = \frac{b}{c}$

$b^2 = ac$

$a^2 + ac = c^2$

$a(a+c) = c^2$

$a^2 - c^2 = -ac$

$(a-c)(a+c) = -ac$

$4^2 + (4-x)^2 = (4+x)^2$

$16 + \cancel{x^2} - 8x + \cancel{16} = \cancel{x^2} + 8x + \cancel{16}$

$16 = 16x$

$x = 1$

Math 9H Section 2.2 Zero and Negative Exponents

Name: Claire Date: 10.2

1. Simplify each of the following expressions without a calculator:

a) 5^{-1} $\quad \frac{1}{5}$

b) $(-5)^{-2}$ $\quad \frac{1}{25}$

c) 223^{0} $\quad 1$

d) $(0.2)^{-3}$ $\quad 125$

e) $(0.5)^{-5}$ $\quad 32$

f) $(-a)^{-3}$ $\quad -\frac{1}{a^3}$

g) $(-3b)^{-3}$ $\quad -\frac{1}{27b^3}$

h) $(4a^3b^2)^{-2}$ $\quad \frac{1}{16a^6b^4}$

i) $-(-2a^{-1})^{-4}$ $\quad -\frac{1}{16\frac{1}{a^4}} = -\frac{a^4}{16}$

j) $-(-3ab^{-1})^{-3}$ $\quad \frac{b^3}{27a^3}$

2. Express each of the following as a common fraction:

a) $\left(\frac{2}{3}\right)^{-2}$ $\quad \frac{9}{4}$

b) $\left(-\frac{4}{3}\right)^{-2}$ $\quad \frac{9}{16}$

c) $\left(\frac{1}{3}\right)^{-3}$ $\quad 27$

d) $\left(\frac{2}{3}+\frac{3}{2}\right)^{-2}$ $\quad = \left(\frac{13}{6}\right)^{-2} = \frac{36}{169}$

e) $\left(\frac{1}{2}+\left(\frac{1}{3}\right)^{-1}\right)^{-2}$ $\quad = \left(\frac{7}{2}\right)^{-2} = \frac{4}{49}$

f) $\left(\frac{1}{6}-\frac{1}{12}\right)^{-2}$ $\quad = \left(\frac{1}{12}\right)^{-2} = 144$

g) $\left(5^{-2}-5^{-1}\right)^{-2}$ $\quad = \left(\frac{1}{25}-\frac{1}{5}\right)^{-2} = \left(-\frac{4}{25}\right)^{-2} = \frac{625}{16}$

h) $-\left(\frac{-a}{b}\right)^{-4}$ $\quad = -\frac{b^4}{a^4}$

i) $\left(\frac{b}{a}-\frac{a}{b}\right)^{-1}$ $\quad = \left(\frac{b^2-a^2}{ab}\right)^{-1} = \frac{ab}{b^2-a^2}$

j) $\left(\frac{a}{b}-\frac{b}{a}\right)^{-2}$ $\quad = \left(\frac{a^2-b^2}{ab}\right)^{-2} = \frac{a^2b^2}{a^4-2a^2b^2+b^4}$

3. Express in simplest form:

a) $\frac{3^{-2}+4^{-1}}{5^{-1}}$ $\quad = \frac{\frac{1}{9}+\frac{1}{4}}{\frac{1}{5}} = \frac{\frac{13}{36}}{\frac{1}{5}} = \frac{65}{36}$

b) $\frac{2(3)^{-2}+3(2)^{-3}}{2(3)^{-2}-3(2)^{-3}}$ $\quad = \frac{\frac{2}{9}+\frac{3}{8}}{\frac{2}{9}-\frac{3}{8}} = \frac{43}{72}\cdot\frac{72}{11} = -\frac{43}{11}$

c) $\frac{(9-6)!\ +\ 3^{-2}}{(10-7)!}$ $\quad = \frac{3!+\frac{1}{9}}{3!} = 1+\frac{1}{9}\cdot\frac{1}{6} = \frac{55}{54}$

d) $\left[4-3(6-8)^{-1}\right]^{-2}$ $\quad = \left(4+\frac{3}{2}\right)^{-2} = \left(\frac{11}{2}\right)^{-2} = \frac{4}{121}$

e) $\frac{5^{-2}\times 4^{-2}}{20^{-2}}$ $\quad = \frac{(5\times4)^{-2}}{20^{-2}} = \frac{20^{-2}}{20^{-2}} = 1$

f) $\left(\frac{1}{ab^2}\right)^{-3}$ $\quad = a^3b^6$

g) $\left(\frac{-2x^2}{3y}\right)^{-3} \div \left(\frac{3}{x}\right)$ $\quad = \frac{27y^3}{-8x^6}\times\frac{x}{3} = -\frac{9y^3}{8x^5}$

h) $\frac{ab^{-2}+ba^{-1}}{ab^{-3}}$ $\quad = \frac{ab^{-2}}{ab^{-3}}+\frac{a^{-1}b}{ab^{-3}} = b+a^{-2}b^4 = \frac{b^4}{a^2}+b$

i) $(ab)^{-2}+b^{-2}$ $\quad = \frac{1}{a^2b^2}+\frac{1}{b^2} = \frac{1}{b^2}\left(\frac{1}{a^2}+1\right) = \frac{1}{b^2}\cdot\frac{1+a^2}{a^2} = \frac{a^2+1}{a^2b^2}$

j) $\frac{a^{-3}+b^{-3}}{ab^{-3}}$ $\quad = \frac{a^{-3}}{ab^{-3}}+\frac{b^{-3}}{ab^{-3}} = a^2b^3+\frac{1}{a} = \frac{a^3b^3+1}{a}$

4. Solve for 'x' in each of the following equations:

a) $\left(\frac{1}{3^4}\right)^{-2} = 9^x$

$3^8 = 9^x$

$9^4 = 9^x$

$x = 4.$

b) $\left(\frac{-1}{2}\right)^{-5} = (-2)^x$

$-2^5 = (-2)^x$

$x = 5.$

c) $\left(\frac{1}{4^{-2}}\right)^{-3} = 16^x$

$\frac{1}{4^6} = 4^{2x}$

$1 = 4^{2x+6}$

$2x+6 = 0$

$x = -3.$

d) $\left(\frac{1}{4}\right)^{-2} = 8^{-x}$

$4^2 = 8^{-x}$

$2^4 = 2^{-3x}.$

$-3x = 4$

$x = -\frac{4}{3}.$

e) $\left(3^{14}\right)^{-2} = \left(\frac{1}{9}\right)^x$

$\frac{1}{3^{28}} = \frac{1}{9^x}$

$\frac{1}{3^{28}} = \frac{1}{3^{2x}}$

$x = 14.$

f) $\left(\frac{3^8}{2^4}\right)^{x-2} = 1$

$3^{8x-16} = 2^{4x-8}$

$=$

g) $2^{-x} + 2^{-x} = 8$

$2 \times 2^{-x} = 8$

$2^{-x+1} = 8^3.$

$-x+1 = 3$

$x = -2.$

h) $(5)^{-2} = (125)^x$

$5^2 = 5^{3x}$

$1 = 5^{3x+2}.$

$3x+2 = 0$

$x = -\frac{2}{3}.$

i) $2^{-3} - 8^{-1} = 2^x$

$\frac{1}{2^3} - \frac{1}{8} = 2^x$

$0 = 2^x.$

j) $3^{-3} - 3^{-2} = 3^x$

$\frac{1}{3^3} - \frac{3}{3^3} = 3^x$

$-\frac{2}{3^3} = 3^x$

$-2 = 3^{x+3}$

5. List the following from the least to the greatest:

i) $\left(\frac{1}{8}\right)^{-3}$, ii) $\left((-2)^2\right)^3$, iii) $\left(-\frac{1}{2}\right)^{-2}(-2)^{-2}$, iv) $(-2)^{-2}\left(\left((2)^{-1}\right)^{-2}\right)^{-3}$

$i < iv < iii < ii$

$\frac{1}{8^3} = \frac{1}{2^9}$

$= 2^6.$

$= 2^2 \cdot \frac{1}{2^2}$

$= 1.$

$= \frac{1}{2^2} \cdot 2^{-6}$

$= \frac{1}{2^8}$

$\frac{1}{2^9} < \frac{1}{2^8} < 1 < 2^6.$

6. If $\frac{1}{2x+1} = 3^{-1}$, then what is the value of $2x+8$?

$\frac{1}{2x+1} = \frac{1}{3}.$

$2x+1 = 3$

$x = 1$

$2x+8 = \boxed{10}$

7. If $4^{-3} + 4^{-3} + 4^{-3} + 4^{-3} = 2^x$, then what is the value of 'x'?

$4 \times 4^{-3} = 2^x$

$4^{-2} = 2^x$

$2^{-4} = 2^x$

$x = \boxed{-4}$

8. What is the reciprocal of $\left(\frac{2}{3} + \frac{3}{2}\right)^{-3}$. Express your answer as a common fraction.

$= \left(\frac{13}{6}\right)^{-3}$

$= \frac{6^3}{13^3}.$

9. Express $\left(1^{-1}+2^{-1}+3^{-1}+4^{-1}+5^{-1}+6^{-1}\right)^{-1}$ as a common fraction in simplist term.

$$=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)^{-1}$$

$$=\left(\frac{60+30+20+15+12+10}{60}\right)^{-1}=\boxed{\frac{60}{147}}//$$

10. Solve for x: $2+\left(2+\left(2+(2)^{-1}\right)^{-1}\right)^{-1}=x$. Express your answer as a common fraction.

$$x=2+\left(2+\left(2+\frac{1}{2}\right)^{-1}\right)^{-1}$$

$$=2+\left(2+\frac{2}{5}\right)^{-1}$$

$$=2+\frac{5}{12}$$

$$=\boxed{\frac{29}{12}}//$$

11. Given that the positive integers 'a','b',and 'c' satisfy the equation: $\frac{4}{5}=a^{-1}+b^{-1}+c^{-1}$. What is the largest value of $a+b+c$?

$$\frac{4}{5}=\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\frac{1}{4\times5} \qquad \frac{4}{5}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$$

$$=\frac{1}{2}+\underbrace{\frac{1}{6}+\frac{1}{12}}+\frac{1}{20}$$

$$=\frac{1}{2}+\frac{1}{4}+\frac{1}{20} \qquad \begin{cases} a=2 \\ b=4 \\ c=20 \end{cases} \Rightarrow a+b+c=\boxed{26}//$$

12. For what values of "x" will the expression be true? $4\left(4^2+4^2+4^2+4^2\right)=2^x$

$$4\times4\times4^2=2^x$$

$$2^8=2^y$$

$$x=\boxed{8}//$$

13. Jason entered a positive number onto his calculator and pressed the x^2 button and got the value $0.\overline{012345679}$. What value would he have gotten if he had pressed the $\sqrt{x}$ button instead? Try to do this question without a calculator.

Let $\sqrt{x}=a$, $x=a^2$, $x^2=a^4$.

$$x^2=a^4=0.\overline{012345679}$$

$$a=\sqrt[4]{0.\overline{012345679}}//$$

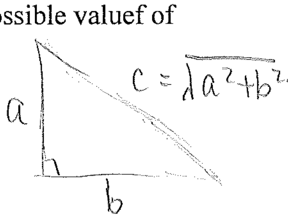
14. Challenge: The posistive integers a,b,and c satisfy $a^{-2}+b^{-2}=c^{-2}$. What is the sum of all possible valuef of "a' such that $a\le100$?

$$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$$

$$\frac{a^2+b^2}{a^2b^2}=\frac{1}{c^2}$$

$$c^2=\frac{a^2b^2}{a^2+b^2}$$

$$c=\frac{ab}{\sqrt{a^2+b^2}}=\frac{ab}{c}$$



15. The value of $A+B$ that satisfies $(6^{30}+6^{-30})(6^{30}-6^{-30})=3^{A}8^{B}-3^{-A}8^{-B}$ is:

(a) 30 (b) 40 (c) 60 (d) 80 (e) 120

$$(6^{30})^2-(6^{-30})^2=3^A8^B-\frac{1}{3^A8^B}$$

$$6^{60}-\frac{1}{6^{60}}=3^A8^B-\frac{1}{3^A8^B}$$

$$\therefore 3^A8^B=6^{60}$$

$$3^A8^B=3^{60}\cdot 8^{20}$$

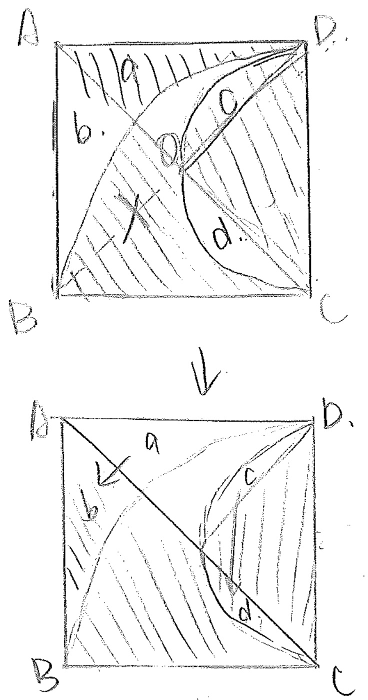
$$\therefore A=60,\ B=20.\ \therefore A+B=80$$

16. For what value of n does $(2^{2007}-2^{2006})(2^{1997}-2^{1996})=2^{n}$?

$$2^{2006}(2-1)\cdot 2^{1996}(2-1)=2^n$$

$$2^{2006+1996}=2^n$$

$$n=\boxed{4002}$$



$\because a=b,\ c=d.$

$\therefore$ Shaded area $= a+X+c+S_{\triangle COD}$

$= \underbrace{b+X+d}+S_{\triangle COD}$

$= S_{\triangle ABC}+S_{\triangle COD}$

$= 3S_{\triangle COD}$

$= \frac{3}{4}S_{ABCD}$

$= \frac{3}{4}\times 8$

$= \boxed{6}$

Name: Claire Date: 10.4.

Math 9H Section 2.3 Fractional Exponents:

1. Simplify each of the following expressions:

$a)\ 16^{\frac{1}{2}}$ 4

$b)\ 100^{\frac{3}{2}}$ 1000.

$c)\ 64^{\frac{1}{3}}$ 4.

$d)\ 27^{\frac{2}{3}}$ 9.

$e)\ (-27)^{\frac{1}{3}}$ -3.

$f)\ 49^{\frac{3}{2}}$ 7.

$g)\ (-1000)^{\frac{2}{3}}$ 100.

$h)\ \left(16^{\frac{1}{2}}\right)^{\frac{3}{2}}$ 8.

$i)\ \left(1000^{\frac{2}{3}}\right)^{\frac{1}{2}}$ 10.

$j)\ \left(9^{\frac{1}{2}}\right)^{\frac{5}{2}}$ $\sqrt{243}$

$k)\ \left(81^{\frac{3}{4}}\right)^{\frac{1}{2}}$ $\sqrt{27}$

$l)\ \left(-64^{\frac{4}{3}}\right)^{\frac{1}{2}}$ 16.

2. Express the following as a single power (Simplify or reduce the base if possible)

a) $\sqrt{15}$ $15^{\frac{1}{2}}$.

b) $\sqrt[3]{20}$ $20^{\frac{1}{3}}$.

c) $27^{0.\bar{3}}$ $= 27^{\frac{1}{3}}$. 3.

d) $32^{1.2}$ $= 32^{\frac{6}{5}}$ 64.

e) $\sqrt[3]{-9}$ $(-9)^{\frac{1}{3}}$.

f) $\sqrt[3]{111}$ $111^{\frac{1}{3}}$.

g) $\sqrt[3]{81}$ $81^{\frac{1}{3}}$.

h) $\sqrt[3]{128}$ $4\sqrt[3]{2}$.

$i)(-216)^{\frac{2}{5}}$ $\sqrt[5]{216^2}$.

j) $\sqrt[5]{(32)^3}$ 8.

$k)\ (36)^{-\frac{7}{5}}$ $\sqrt[5]{-36^7}$

$l)\ \dfrac{1}{(-8)^{-\frac{2}{3}}}$ $1 \div \frac{1}{4}$ 4.

m) $\left(\dfrac{1}{16}\right)^{\frac{-1}{2}}$ $= 16^{\frac{1}{2}}$ 4

n) $\left(-\dfrac{1}{27}\right)^{\frac{-2}{3}}$ $\sqrt[3]{(-27)^2} = 9$.

p) $\left(\dfrac{8}{125}\right)^{\frac{2}{3}}$ $= \left(\frac{2}{5}\right)^2$ $\frac{4}{25}$.

q) $\left(\dfrac{9}{16}\right)^{\frac{-3}{2}}$ $= \left(\frac{4}{3}\right)^3$. $\frac{64}{27}$.

3. Simplify the following:

$a)\ x^2 \times x^{\frac{1}{2}}$ $x^{\frac{5}{2}}$.

$b)\ x^2 \times x^{\frac{1}{2}}$ $x^{\frac{5}{2}}$.

$c)\ x^{\frac{3}{2}} \times x^{\frac{4}{2}} \times x^{\frac{5}{2}} \div x^{\frac{6}{2}}$ x^3.

$d)\ x^{\frac{3}{2}} \div x^{\frac{4}{3}}$ $= x^{\frac{9}{6} - \frac{8}{6}}$ $x^{\frac{1}{6}}$.

$e)\ \left(\dfrac{1}{x}\right)^{\frac{3}{2}} \div \sqrt[5]{x^4} \div \left(\dfrac{-1}{x^2}\right)^{\frac{5}{3}}$

$= x^{-\frac{3}{2}} \div x^{\frac{4}{5}} \div \left(-\frac{1}{x^{\frac{10}{3}}}\right)$

$= -x^{-\frac{3}{2} - \frac{4}{5} + \frac{10}{3}}$.

$= -x^{\frac{31}{30}}$

$f)\ \sqrt{\left(\sqrt{y}\right)^{\frac{3}{2}}(-y)^{\frac{2}{3}}}$

$= \sqrt{y^{\frac{3}{4} + \frac{2}{3}}}$

$= y^{\frac{17}{24}}$

$g)\ \sqrt[3]{\dfrac{-8}{x}}$ $-\frac{2}{x^{\frac{1}{3}}}$

$h)\ \left(x^2 \times x^{\frac{1}{2}}\right)^{\frac{2}{3}} \div x^{\frac{9}{4}}$

$= x^{\frac{5}{2} \cdot \frac{2}{3}} \div x^{\frac{9}{4}}$.

$= x^{\frac{20}{12} - \frac{27}{12}}$

$= x^{\frac{-7}{12}}$

4. Simplify each expression and place them in increasing order: A < C < B.

$A = 256^{\frac{-5}{8}}$ $= \left(\frac{1}{2}\right)^5$ $= \frac{1}{32}$.

$B = \left(81^{\frac{1}{4}}\right)^3$ $= (3)^3$ $= 27$.

$C = \left(\frac{2}{5}\right)^{-3}$ $= \frac{125}{8}$

$D = \cancel{0}49^{\frac{3}{2}}$ X.

5. Simplify the following:

a) $125^{\frac{2}{3}} \div \left(\sqrt[5]{32}\right)^6 - \sqrt{\sqrt{\frac{1}{256}}}$
$= 25 \div 64 - \frac{1}{4}$.
$= \frac{25-16}{64} = \frac{9}{64}$

b) $\sqrt{\left(\sqrt[3]{64}\right)^2 + \left(\sqrt{81}\right)^{\frac{1}{2}}}$
$= \sqrt{16+3}$
$= \sqrt{19}$.

c) $\left(\sqrt{\sqrt{256}}\right)^{-3} - \left(\frac{1}{-27}\right)^{\frac{1}{3}} \times \left(\sqrt[3]{216}\right)$
$= \frac{1}{4^3} + \frac{1}{3} \times 6^2$
$= 2\frac{1}{64}$.

d) $\left(32^{0.2} + 27^{-0.\bar{3}} \times 216^{0.\bar{6}}\right)^{-2}$
$= \left(5 + \frac{1}{3} \times 36\right)$
$= \frac{1}{360}$.

e) $\sqrt{(100)^{-1.5} \times (27)^{-1.\bar{3}} \div (6.25)^{-0.5}}$
$= \sqrt{\frac{1}{1000} \times \frac{1}{81} \times \frac{5}{2}}$
$= \sqrt{\frac{1}{2^2 \times 10^2 \times 9^2}} = \frac{1}{180}$

6. The population of a cockroach colony can be given by the formula: $F = I \times N^{\frac{A}{L}}$. "F" is the final population, "I" is the initial population, "A" is the time to reach the final population, "N" is the rate of growth, and "L" is the time for one cycle of growth to occur. If a colony of 20 cockroaches increases by 8 times every 3 six days, what is the final population after 7 days?

$F = 20 \times 8^{\frac{7}{3}}$
$= 20 \times 128 = \boxed{2560}$ cockroaches.

7. If the final population of the cockroaches is 2162688 and the initial population was 33, how many days did it take the colony to grow?

$2162688 = 33 \times 8^{\frac{L}{3}}$
$65536 = 2^L$
$2^L = 256^2$
$2^L = 2^{16}$
$L = \boxed{16}$

8. If the accumulate amount of an investment is given by the formula: $A = 3000(1.21)^t$, where "t" is the number of years. How much will be accumulated after 3.5 years?

$A = 3000 \times (1.1^2)^{3.5}$
$= 3000 \times 1.1^7$

9. If a, b, and c are distinct positive integers such that $abc = 16$ then what is the largest possible value of: $a^b - b^c + c^a$?

$1 \times 2 \times 8 = 16$.

$8^1 - 1^2 + 2^8 = \boxed{263}$

10. The year 2002 is a palindromic number, i.e., it reads the same forwards and backwards. The number of years in the third millenium, i.e., between the years 2001 and 3000, that have this property is:

(a) 1 (b) 10 (c) 11 (d) 99 (e) 100

20, 21, 22, 23, 24, 25, 26, 27, 28, 29 → 10.

11. Positive integers which read the same backwards as forwards are called *palindromes*; for example, 11, 252, and 31013 are palindromes. The number of palindromes less than 10^6, but greater than 10 are:

100,000.

(a) 999 (b) 10^3 (c) 1089 (d) 1989 (e) 2209

2 digits - 9. 4 digits - 90. 6 digits - 900. $9+90+900+90+900=1989$

3 digits 90. 5 digits - 900

12. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2882 is a four-digit palindrome and 49194 is a five digit palindrome. There are pairs of four-digit palindromes whose sum is a five digit palindrome. One such pair is 2882 and 9339. How many such pairs are there?

digits number add up to 11: 2+9, 3+8, 4+7, 5+6

1 _ _ _ _ (1). Ex: 2aa2 + 9bb9 = 1 _ _ _ 1 $4\times(4+1)=20$ pairs.

has to be 1.

11011 + 110(a+b) = 1 _ _ _ 1

13. $\frac{1}{2}+\frac{2}{2^2}+\frac{2^2}{2^3}+\frac{2^3}{2^4}+\ldots\frac{2^{2002}}{2^{2003}}+\frac{2^{2003}}{2^{2004}}$ is equal to: a+b = 11 or 0.

(A) 1002 (B) 501 (C) $\frac{1}{2^{2004}}$ (D) 2004 (E) $\frac{2004}{2^{2004}}$

$2004\times\frac{1}{2}=1002.$

14. 10^{100} is known as a googol. What is 1000^{100} equal to?

$=(10\times10\times10)^{100}=(10^{100})^3.$

(A) 100 googol (B) 3 googol (C) $\text{googol}^{\text{googol}}$

(D) googol^2 (E) googol^3

15. If "x' is an integer less than 100, then how many different values of "x" will make the expression $\sqrt{1+2+3+4+x}$ an integer? (list out all the values for "x")

(A) 6 $\sqrt{10+x}$ (B) 7 (C) 8 (D) 9 (E) 10

x = 6, 15, 26, 39, 54, 71, 90

↑ ↑ ↑ ↑ ↑ ↑ ↑

$\sqrt{4^2}$ $\sqrt{5^2}$ $\sqrt{6^2}$ $\sqrt{7^2}$ $\sqrt{8^2}$ $\sqrt{9^2}$ $\sqrt{10^2}$

16. A positive integer is called "multiplicatively perfect" if it is equal to the product of its proper divisors. For example, 10 is a multiplicatively perfect since its proper divisors are 1, 2, and 5 and it is true that $1\times2\times5=10$. How many multiplicatively perfect integers are there between 2 and 100?

2, 3, 5, 7, 11, 13, 17, 23, 29, 31, 37, 41, 43, 47.

① $2\times\square$ ② $3\times\square$ ③ $5\times\square$ ④ $7\times\square$ $13+8+4+2=27$

13 8 4 2

17. What is the smallest positive integer c for which $\sqrt{a}+\sqrt{b}=\sqrt{c}$ has a solution in unequal positive integers?

a=1, b=4 → $1+2=\sqrt{c}$

$3=\sqrt{c}$

$9=c$

Name: Claire Li　　　　　　　　　　　　　　　　Date: 10. 8.

Math 9H Section 2.5 Operations with Radicals

1. Simplify each of the following expressions by multiplication or division:

$a)\sqrt{12}\times\sqrt{24}$ $= 2\sqrt{3}\times 2\sqrt{6}.$ $= 12\sqrt{2}.$	$b)\sqrt{8}\times\sqrt{18}$ $2\sqrt{2}\times 3\sqrt{2}$ 12	$c)2\sqrt{5}\times 3\sqrt{2}$ $= 6\sqrt{10}.$
$d)-8\sqrt{6}\times 3\sqrt{8}$ $= -8\times 3\times 4\sqrt{3}.$ $= -96\sqrt{3}.$	$e)11\sqrt{3}\times 4\sqrt{27}$ $= 44\times 9.$ $= 396.$	$f)\sqrt{20}\times\sqrt{32}\times\sqrt{18}\times\sqrt{125}$ $= \sqrt{4\times 5\times 5\times 25\times 4^2\times 2\times 9}.$ $= 1200.$
$g)0.8\sqrt{80}\times 0.125\sqrt{90}\times 0.5\sqrt{50}$ $= \frac{4}{5}\times\frac{1}{8}\times\frac{1}{2}\times\sqrt{400\times 2\times 9\times 25\times 2}.$ $= \frac{1}{20}\times 20\times 2\times 3\times 5 = 30$	$h)5\sqrt{3d}\times 2\sqrt{2d}\times 3\sqrt{3d}$ $= 30\times 3d\sqrt{2d}.$ $= 90d\sqrt{2d}.$	$i)7\sqrt{6a}\times 3\sqrt{2b}\times 2\sqrt{3ab}$ $= 42\sqrt{6^2\cdot a^2\cdot b^2}.$ $= 252ab.$

2. Simplify each of the following expressions by addition or subtraction

$a)5\sqrt{7}-3\sqrt{7}+6\sqrt{7}$ $= 8\sqrt{7}.$	$b)2\sqrt{10}+7\sqrt{10}-6\sqrt{10}$ $= 3\sqrt{10}$	$c)\sqrt{12}-2\sqrt{75}$ $= 2\sqrt{3}-10\sqrt{3}$ $= -8\sqrt{3}.$
$d)\sqrt{54}+\sqrt{150}-2\sqrt{216}$ $= 3\sqrt{6}+5\sqrt{6}-12\sqrt{6}.$ $= -4\sqrt{6}$	$e)\sqrt{48}-\frac{2}{3}\sqrt{20}-0.5\sqrt{27}+2\sqrt{45}$ $= 4\sqrt{3}-\frac{4}{3}\sqrt{5}-\frac{3}{2}\sqrt{3}+6\sqrt{5}$ $= \frac{5}{2}\sqrt{3}+\frac{14}{3}\sqrt{5}$	$f)\frac{2}{5}\sqrt{125}-\frac{2}{3}\sqrt{243}-\frac{1}{3}\sqrt{45}+\frac{1}{2}\sqrt{48}$ $= 2\sqrt{5}-6\sqrt{3}-\sqrt{5}\cdot 2\sqrt{3}.$ $= \sqrt{5}-8\sqrt{3}.$
$g)\frac{2}{3}\sqrt{72}-0.\dot{3}\sqrt{50}+0.\overline{285714}\sqrt{98}$ $= 4\sqrt{2}-\frac{5}{3}\sqrt{2}+\frac{2}{7}7\sqrt{2}.$ $= \frac{13}{3}\sqrt{2}.$		$h)\frac{\sqrt{8a}}{4}+\frac{\sqrt{27b}}{3}-0.\dot{3}\sqrt{50a}-4\sqrt{75b}$ $= \frac{\sqrt{2a}}{2}+\sqrt{3b}-\frac{5}{3}\sqrt{2a}-20\sqrt{3b}.$ $= -\frac{7}{6}\sqrt{2a}-19\sqrt{3b}$

3. Expand and simplify each of the following expressions:

$a)\sqrt{2}\left(\sqrt{5}-\sqrt{7}\right)$ $= \sqrt{10}-\sqrt{14}$	$b)2\sqrt{5}\left(3\sqrt{2}+4\sqrt{3}\right)$ $= 6\sqrt{10}+8\sqrt{15}$	$c)6\sqrt{6}\left(3\sqrt{2}-4\sqrt{3}\right)$ $= 36\sqrt{3}-72\sqrt{2}.$
$d)-4\sqrt{3}\left(2\sqrt{5}-6\sqrt{6}-3\sqrt{12}\right)$ $= -8\sqrt{15}+72\sqrt{2}+72.$	$e)2\sqrt{3}\left(5\sqrt{8}+2\sqrt{3}-4\sqrt{5}+3\sqrt{6}\right)$ $= 10\sqrt{24}+12-8\sqrt{15}+18\sqrt{2}.$ $= 12+20\sqrt{6}-8\sqrt{15}+18\sqrt{2}.$	$f)\left(\sqrt{8}-5\right)\left(\sqrt{2}-3\right)$ $= 4-3\sqrt{8}-5\sqrt{2}-15.$ $= -11-11\sqrt{2}.$
$g)\left(3-\sqrt{2}\right)^2$ $= 9-6\sqrt{2}+2.$ $= 11-6\sqrt{2}.$	$h)\left(4\sqrt{2}-2\sqrt{3}\right)\left(4\sqrt{2}+2\sqrt{3}\right)$ $= (4\sqrt{2})^2-(2\sqrt{3})^2$ $32-12$ $= 20.$	$i)\left(6\sqrt{8}-3\sqrt{2}\right)\left(6\sqrt{8}+3\sqrt{2}\right)$ $= (6\sqrt{8})^2-(3\sqrt{2})^2$ $= 288-18$ $= 270.$

4. Rationalize each of the following expressions:

a) $\frac{\sqrt{24}}{\sqrt{3}}$

$= 2\sqrt{2}$.

b) $\frac{3\sqrt{20}}{2\sqrt{10}}$

$= \frac{3\sqrt{2}}{2}$

c) $\frac{3\sqrt{18}}{5\sqrt{24}}$

$= \frac{9\sqrt{2}}{10\sqrt{6}}$

$= \frac{9}{10\sqrt{6}} = \frac{9\sqrt{6}}{60} = \frac{3\sqrt{3}}{20}$.

d) $\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{3}}$

$= \frac{\sqrt{5}}{5} - \frac{\sqrt{3}}{3}$.

$= \frac{3\sqrt{5}+5\sqrt{3}}{15}$

e) $\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{6}}$

$= \frac{\sqrt{3}}{3} + \frac{8\sqrt{6}}{6\cdot 3}$.

$= \frac{\sqrt{3}+\sqrt{6}}{3}$.

f) $\frac{5}{\sqrt{5}} - \frac{8}{\sqrt{2}}$

$= \frac{5\sqrt{5}}{5} - \frac{8\sqrt{2}}{2}$

$= \sqrt{5} - 4\sqrt{2}$.

g) $\frac{3\sqrt{48}}{2\sqrt{75}} - \frac{2\sqrt{24}}{\sqrt{96}}$

$= \frac{12\sqrt{3}}{10\sqrt{3}} - \frac{\cancel{x}}{\cancel{\sqrt{x}}}$

$= \frac{6}{5} - 1 = \frac{1}{5}$

h) $\frac{3\sqrt{5}}{\sqrt{20}} + \frac{4\sqrt{3}}{\sqrt{27}}$

$= \frac{3}{2} + \frac{4}{3}$

$= \frac{17}{6}$.

i) $\frac{2\sqrt{3}}{\sqrt{9}} - \frac{3\sqrt{5}}{\sqrt{125}}$

$= \frac{2\sqrt{3}}{3} - \frac{3}{5}$.

$= \frac{10\sqrt{3}-9}{15}$

5. If $A = 2\sqrt{6}$ and $B = 6\sqrt{2}$ find each of the following:

a) $A \times B$

$= 2\sqrt{6} \times 6\sqrt{2}$.

$= 24\sqrt{3}$.

b. $2AB - (AB)^2$

$= 2\times 24\sqrt{3} - (24\sqrt{3})$

$= 48\sqrt{3} - 1728$.

c) $\frac{A^2 - B^2}{3A}$

$= \frac{(2\sqrt{6})^2 - (6\sqrt{2})^2}{3\cdot 2\sqrt{6}}$

$= \frac{24 - 72}{6\sqrt{6}}$

$= -\frac{8}{\sqrt{6}} = -\frac{8\sqrt{6}}{6}$

$= -\frac{4\sqrt{6}}{3}$

6. What is $\sqrt{6} \times \sqrt[3]{2}$ equal to? (Justify your answer:) Choose one:

$6^{\frac{1}{2}} = 6^{\frac{3}{6}}$ $2^{\frac{1}{3}} = 2^{\frac{2}{6}}$

$\sqrt[6]{6^3} \times \sqrt[6]{2^2} = \boxed{\sqrt[6]{864}}$

a) $\sqrt[3]{12}$ b) $\sqrt{12}$ c) $\sqrt[6]{12}$ d) Can't multiply them

7. Find the unknown value "K" in each of the following expressions:

a) $K \times 3\sqrt{24} = 2\sqrt{3} \times 6\sqrt{10}$

$K \times 6\sqrt{6} = 12\sqrt{6} \times \sqrt{5}$.

$K = \boxed{2\sqrt{5}}$

Answer:__________

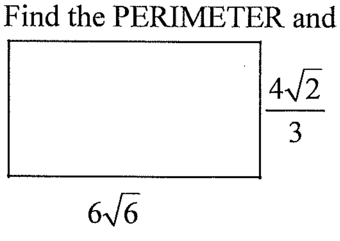
b) $8\sqrt{3} = \frac{4\sqrt{48}}{\sqrt{K}}$

$2\sqrt{3}K = \sqrt{48}$, $\sqrt{K} = 2$

$2\sqrt{3}\cdot\sqrt{K} = 4\sqrt{3}$

Answer: $K = 4$

8. Find the PERIMETER and AREA of the following rectangle:



$P = (6\sqrt{6} + \frac{4\sqrt{2}}{3}) \times 2$

$= 12\sqrt{6} + \frac{8}{3\sqrt{2}}$.

$A = \frac{4\sqrt{2}}{3} \times 6\sqrt{6}$.

$= 16\sqrt{3}$.

$P = 12\sqrt{6} + \frac{8}{3\sqrt{2}}$

Answer: $A = 16\sqrt{3}$.

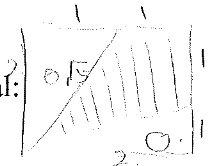
$2x^2+2x-4=0$

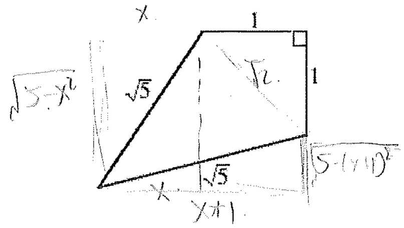
$x^2+x-2=0$

$(x+2)(x-1)=0$

$x=1$

9. Given the following dimensions, find the area of the quadrilateral:





$\sqrt{5-(x+1)^2}+1=\sqrt{5-x^2}$

$5-x^2-2x-1 \quad 5-x^2=2\sqrt{5-x^2}+1$

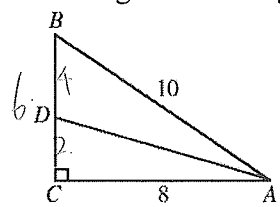
$x+1=\sqrt{5-x^2}$

$x^2+2x+1=5-x^2$

$2^2-2\times1=\boxed{2}$

Answer: 2.

10. Triangle ABC is right-angled with $AB = 10$cm and $AC = 8$cm. If $BC = 3DC$, then AD equals:



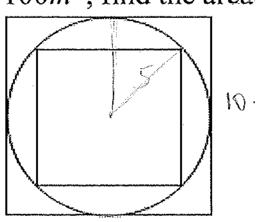
$\sqrt{2^2+8^2}=\sqrt{4+64}$

$\sqrt{68}$

$=2\sqrt{17}$ cm

Answer: $2\sqrt{17}$ cm.

11. A circle is inscribing and circumscribing two separate squares. If the area of the large square is $100m^2$, find the area of the small square:



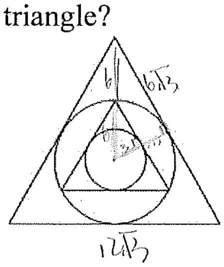
$\left(\frac{5}{\sqrt{2}}\times2\right)^2$

$=(5\sqrt{2})^2$

$=50m^2$

Answer: $50\,m^2$

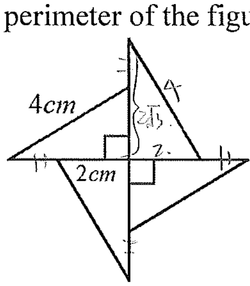
12. A circle is inscribing and circumscribing two separate equilateral triangles. Within the inner triangle, another circle is inscribed. If the radius of the small circle is 3cm, what is the perimeter of the larger triangle?



$12\sqrt{3}\times3=36\sqrt{3}$ cm.

Answer: $36\sqrt{3}$ cm.

13. Each right triangle in the figure shown has a hypotenuse 4cm and the shortest side 2 cm. Find the perimeter of the figure:



$\sqrt{4^2-2^2}=\sqrt{12}=2\sqrt{3}$

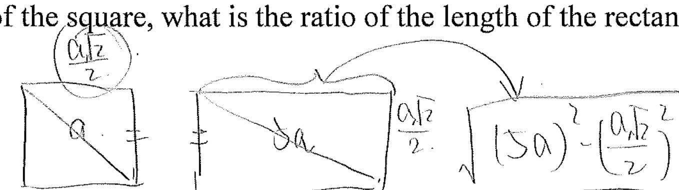
$(2\sqrt{3}-2)\times4+4\times4$

$=(2\sqrt{3}+2)\times4$

$=8\sqrt{3}+8$ cm.

Answer: $(8\sqrt{3}+8)$cm

14. A square and a rectangle have the same width. If the diagonal of the rectangle is 5 times the diagonal of the square, what is the ratio of the length of the rectangle to the length of the square?



$\sqrt{(5a)^2-\left(\frac{a\sqrt{2}}{2}\right)^2}$

$=\sqrt{25a^2-\frac{a^2}{2}}$

$=\frac{7\sqrt{2}a}{2}$

Answer: 1:7.

15. Arrange in order from the least to the greatest:

a) $-6\sqrt{2},\ -3\sqrt{7},\ -2\sqrt{17},\ -4\sqrt{5},\ -2\sqrt{21},\ -5\sqrt{3}$ Answer: ____

$-\sqrt{72}.\quad -\sqrt{63}\quad -\sqrt{68}\quad -\sqrt{80}\quad -\sqrt{84}\quad -\sqrt{75}.$

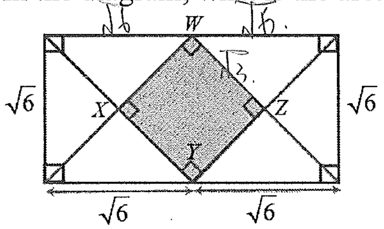
$-2\sqrt{24} < -4\sqrt{5} < -5\sqrt{3} < -6\sqrt{2} < -2\sqrt{17} < -3\sqrt{7}.$

b) $6\sqrt{0.1},\ 3\sqrt{0.7},\ 7\sqrt{0.05},\ 2\sqrt{0.8},\ 4\sqrt{0.5},\ 5\sqrt{0.3}$ Answer: ____

$\sqrt{3.6}\quad \sqrt{6.3}\quad \sqrt{2.45}\quad \sqrt{3.2}\quad \sqrt{8}.\quad \sqrt{7.5}.$

$7\sqrt{0.05} < 2\sqrt{0.8} < 6\sqrt{0.1} < 3\sqrt{0.7} < 5\sqrt{0.3} < 4\sqrt{0.5}.$

16. In the diagram, what is the area of square WXYZ

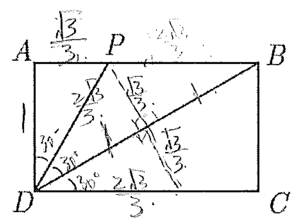


$(\sqrt{3})^2 = 3.$

Answer: 3.

17.

In rectangle $ABCD$, $AD = 1$, P is on $\overline{AB}$, and $\overline{DB}$ and $\overline{DP}$ trisect $\angle ADC$. What is the perimeter of $\triangle BDP$?



(A) $3+\dfrac{\sqrt{3}}{3}$ (B) $2+\dfrac{4\sqrt{3}}{3}$ (C) $2+2\sqrt{2}$

(D) $\dfrac{3+3\sqrt{5}}{2}$ (E) $2+\dfrac{5\sqrt{3}}{3}$

$1+1+\dfrac{2\sqrt{3}}{3}+\dfrac{2\sqrt{3}}{3} = \boxed{2+\dfrac{4\sqrt{3}}{3}}$

18.

Simplify

$$\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\sqrt{x}}}}. = \sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\cdot x^{\frac{1}{2}}}}} = \sqrt[3]{x\sqrt[3]{x\sqrt[3]{x^{\frac{3}{2}}}}}$$

(A) $\sqrt{x}$ (B) $\sqrt[3]{x^2}$ (C) $\sqrt[27]{x^2}$ (D) $\sqrt[54]{x}$ (E) $\sqrt[81]{x^{80}}$

$$= \sqrt[3]{x\sqrt[3]{x\cdot x^{\frac{1}{2}}}}$$

$$= \sqrt[3]{x\cdot x^{\frac{1}{2}}}$$

$$= \sqrt[3]{x^{\frac{3}{2}}}$$

$$= x^{\frac{1}{2}}$$

$$= \sqrt{x}.$$

Name: Claire. Li Date: 10.21

Math 9 Enriched Section 2.6 Rationalizing Radicals

1. Multiply each of the following radicals with its conjugate:

a) $\left(\sqrt{3}+2\right)\left(\sqrt{3}-2\right)$

$=(\sqrt{3})^2-2^2$

$=-1$

b. $\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$

$=5-3$

$=2$

c. $\left(2\sqrt{3}-1\right)\left(2\sqrt{3}+1\right)$

$=12-1$

$=11.$

d. $\left(3-\sqrt{3}\right)\left(3+\sqrt{3}\right)$

$=3^2-3$

$=6$

e. $\left(2\sqrt{3}-2\sqrt{2}\right)\left(2\sqrt{3}+2\sqrt{2}\right)$

$=12-4$

$=8$

f. $\left(2\sqrt{5}-6\right)\left(2\sqrt{5}+6\right)$

$20-36$

$=-16.$

2. Rationalize each of the following expressions:

a) $\dfrac{1}{\sqrt{2}-\sqrt{3}}$

$=\dfrac{\sqrt{2}+\sqrt{3}}{2-3}$

$=-\sqrt{2}-\sqrt{3}$

b. $\dfrac{2}{2\sqrt{3}+5}$

$=\dfrac{2(2\sqrt{3}-5)}{12-25}$

$=\dfrac{2(2\sqrt{3}-5)}{13}$ $\quad \dfrac{10-4\sqrt{3}}{13}$

c. $\dfrac{\sqrt{2}}{2\sqrt{3}+\sqrt{5}}$

$=\dfrac{\sqrt{2}(2\sqrt{3}-\sqrt{5})}{12-5}$

$=\dfrac{2\sqrt{6}-\sqrt{10}}{7}$

d. $\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

$=\dfrac{(\sqrt{2}+\sqrt{3})^2}{3-2}$

$=5+2\sqrt{6}$

e. $\dfrac{\sqrt{10}}{\sqrt{8}-\sqrt{6}}$

$\dfrac{\sqrt{10}(\sqrt{8}+\sqrt{6})}{8-6}$

$=\dfrac{4\sqrt{5}+2\sqrt{15}}{2}$

$=2\sqrt{5}+\sqrt{15}$

f. $\dfrac{5\sqrt{3}}{2\sqrt{2}-3\sqrt{3}}$

$=\dfrac{5\sqrt{3}(2\sqrt{2}+3\sqrt{3})}{8-27}$

$=-\dfrac{10\sqrt{6}+45}{19}$

3. Simplify:

a) $\dfrac{7}{\sqrt{20}}-\dfrac{4}{\sqrt{12}}$

$=\dfrac{7}{2\sqrt{5}}-\dfrac{4}{2\sqrt{3}}$

$=\dfrac{7\sqrt{5}}{10}-\dfrac{2\sqrt{3}}{3}$

b. $\dfrac{3\sqrt{48}}{2\sqrt{75}}-\dfrac{2\sqrt{24}}{\sqrt{96}}$

$=\dfrac{12\sqrt{3}}{10\sqrt{3}}-\dfrac{2}{\sqrt{4}}$

$=\dfrac{6}{5}-1=\dfrac{1}{5}$

c. $\dfrac{3}{\sqrt[3]{12}}$

$=\dfrac{3\sqrt[3]{18}}{\sqrt[3]{12}\cdot\sqrt[3]{18}}$

$=\dfrac{3\sqrt[3]{18}}{6}=\dfrac{\sqrt[3]{18}}{2}$

d. $\dfrac{\sqrt{3}}{\sqrt[3]{36}}$

$=\dfrac{\sqrt{3}\cdot\sqrt[3]{6}}{\sqrt[3]{36}\cdot\sqrt[3]{6}}$

$=\dfrac{\sqrt{3}\sqrt[3]{6}}{6}$

e. $\dfrac{x^4+x^2}{\sqrt{x^3}}$

$=\dfrac{(x^4+x^2)\sqrt{x}}{x\sqrt{x}\cdot\sqrt{x}}$

$=\dfrac{x^4\sqrt{x}+x^2\sqrt{x}}{x^2}$

$=x^2\sqrt{x}+\sqrt{x}.$

f. $\dfrac{5\sqrt{2}}{6-3\sqrt{3}}$

$=\dfrac{5\sqrt{2}\cdot(2+\sqrt{3})}{3(2-\sqrt{3})\cdot(2+\sqrt{3})}$

$=\dfrac{10\sqrt{2}+5\sqrt{6}}{3(4-3)}$

$=\dfrac{10\sqrt{2}+5\sqrt{6}}{3}$

g. $\dfrac{2}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

$= \dfrac{2\left[(\sqrt{2}+\sqrt{3})-\sqrt{5}\right]}{\left[(\sqrt{2}+\sqrt{3})+\sqrt{5}\right]\cdot\left[(\sqrt{2}+\sqrt{3})-\sqrt{5}\right]}$

$= \dfrac{2\sqrt{2}+2\sqrt{3}-2\sqrt{5}}{(\sqrt{2}+\sqrt{3})^2-(\sqrt{5})^2}$

$= \dfrac{2\sqrt{2}+2\sqrt{3}-2\sqrt{5}}{2\sqrt{6}}$

$= \dfrac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{6}$

h) $\dfrac{5}{\sqrt{2}+\sqrt{5}-\sqrt{7}}$

$= \dfrac{5\left[(\sqrt{2}+\sqrt{5})+\sqrt{7}\right]}{\left[(\sqrt{2}+\sqrt{5})-\sqrt{7}\right]\cdot\left[(\sqrt{2}+\sqrt{5})+\sqrt{7}\right]}$

$= \dfrac{5\sqrt{2}+5\sqrt{5}+5\sqrt{7}}{(\sqrt{5}+\sqrt{2})^2-7}$

$= \dfrac{5\sqrt{2}+5\sqrt{5}+5\sqrt{7}}{2\sqrt{10}}$

$= \dfrac{2\sqrt{5}+5\sqrt{2}+\sqrt{70}}{4}$

i) $\dfrac{\sqrt{10}}{\sqrt{2}+\sqrt{6}-\sqrt{8}}$

$= \dfrac{\sqrt{10}(\sqrt{2}+\sqrt{6}+\sqrt{8})}{(\sqrt{2}+\sqrt{6})^2-(\sqrt{8})^2}$

$= \dfrac{2\sqrt{5}+2\sqrt{15}+4\sqrt{5}}{4\sqrt{3}}$

$= \dfrac{2\sqrt{15}+2\sqrt{45}+4\sqrt{15}}{12}$

$= \dfrac{\sqrt{15}+\sqrt{45}+2\sqrt{15}}{6}$

4. The area of an equilateral triangle is $9\sqrt{3}\ cm^2$. What is the number of cm in a side of the triangle?

$\dfrac{\sqrt{3}\cdot s^2}{4} = 9\sqrt{3}$

$s^2 = 36$

$s = 6.$

$\boxed{6cm}$

5. Evaluate: $x=\sqrt{20+\sqrt{20+\sqrt{20+\ldots\ldots}}}$

$x^2 = 20+\sqrt{20+\sqrt{20+\cdots}}$

$x^2-20 = \sqrt{20+\sqrt{20+\cdots}}$

$x^2-20 = x$

$x^2-x-20=0$

$(x-5)(x+4)=0$

$x=5$ or -4.

6. The height and base of a right triangle is 6cm and 11cm respectively. What is the length of the altitude of the triangle from the right-angle to the hypotenuse? Express your answer as a radical in simplest form:

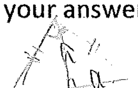
$\sqrt{6^2+11^2} = \sqrt{36+121} = \sqrt{157}$

$\sqrt{157}\cdot h = 6\times 11$

$\sqrt{157}\cdot h = 66$

$h = \boxed{\dfrac{66\sqrt{157}}{157}}$

7. What is the length of the altitude of a right-isosceles triangle from the right-angle to the hypotenuse? $\frac{1}{2}$.
Express your answer as a radical and ratio to the side length (height or base) of the right triangle.



$1 - 1 - \sqrt{2}$

height base hypotenuse

8. For how many real values of x is $\sqrt{120-\sqrt{x}}$ an integer?

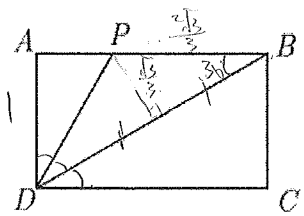
(A) 3 **(B)** 6 **(C)** 9 **(D)** 10 **(E)** 11

$\sqrt{120-\sqrt{x}} = 10, 9, 8, 7, 6, 5, 4, 3, 2, 1$

9. In rectangle $ABCD$, AD=1, P is on $\overline{AB}$, and $\overline{DB}$ and $\overline{DP}$ trisect $\angle ADC$. What is the perimeter of $\triangle BDP$?

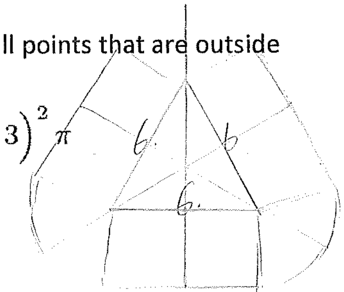
(A) $3+\dfrac{\sqrt{3}}{3}$ (B) $2+\dfrac{4\sqrt{3}}{3}$ (C) $2+2\sqrt{2}$

(D) $\dfrac{3+3\sqrt{5}}{2}$ (E) $2+\dfrac{5\sqrt{3}}{3}$



$\dfrac{2\sqrt{3}}{3}\times 2+2 = \dfrac{4\sqrt{3}}{3}+2.$

10. An equilateral triangle has side length 6. When is the area of the region containing all points that are outside the triangle and not more than 3 units from a point of the triangle?



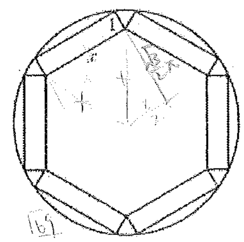
(A) $36+24\sqrt{3}$ (B) $54+9\pi$ (C) $54+18\sqrt{3}+6\pi$ (D) $\left(2\sqrt{3}+3\right)^2\pi$

(E) $9\left(\sqrt{3}+1\right)^2\pi$

$3\times6\times3+3^2\pi$

$=54+9\pi$

11. A round table has radius 4. Six rectangular place mats are placed on the table. Each place mat has width 1 and length x as shown. They are positioned so that each mat has two corners on the edge of the table, these two cormers beimg end points of the same side of length x. Further, the mats ar positioned so that the inner corners each touch an inner corner of an adjacent mat. What is x?



(A) $2\sqrt{5}-\sqrt{3}$ (B) 3 (C) $\dfrac{3\sqrt{7}-\sqrt{3}}{2}$ (D) $2\sqrt{3}$

(E) $\dfrac{5+2\sqrt{3}}{2}$

$(\frac{\sqrt{3}}{2}x+1)^2+(\frac{x}{2})^2=4^2$

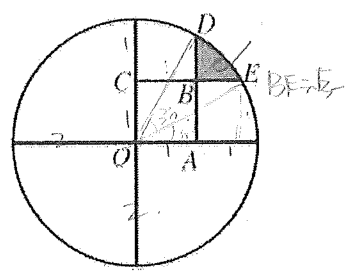
$\frac{3}{4}x^2+\frac{x^2}{4}+\sqrt{3}x+1=4^2$

$x^2+\sqrt{3}x-15=0$

$x=\frac{-\sqrt{3}\pm3\sqrt{7}}{2}$

$x=\frac{3\sqrt{7}-\sqrt{3}}{2}$

12. A circle of radius 2 is centered at O. Square $OABC$ has side length 1. Sides $\overline{AB}$ and $\overline{CB}$ are extended past B to meet the circle at D and E, respectively. What is the area of the shaded reion in the figure, which is bounded by $\overline{BD}$, $\overline{BE}$, and the minor arc connecting D and E?



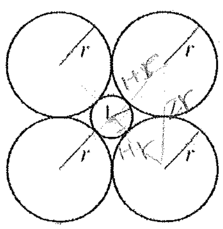
(A) $\frac{\pi}{3}+1-\sqrt{3}$ (B) $\frac{\pi}{2}(2-\sqrt{3})$ (C) $\pi(2-\sqrt{3})$ (D) $\frac{\pi}{6}+\frac{\sqrt{3}-1}{2}$

(E) $\frac{\pi}{3}-1+\sqrt{3}$

$\frac{1}{12}\pi\cdot2^2-\frac{(\sqrt{3}-1)\times1}{8}\times8$

$=\frac{\pi}{3}-\sqrt{3}+1$

13. A cirlce of radius 1 is surrounded by 4 circles of radius r as shown. What is r?



(A) $\sqrt{2}$ (B) $1+\sqrt{2}$ (C) $\sqrt{6}$ (D) 3 (E) $2+\sqrt{2}$

$\sqrt{2}(1+r)=2r$

$1+r=\sqrt{2}r$

$\frac{1}{\sqrt{2}-1}=r$

$\sqrt{2}+1=r$

14. Which of the following is equivalent to $\sqrt{\dfrac{x}{1-\dfrac{x-1}{x}}}$ when $x<0$?

$\sqrt{\dfrac{x}{\frac{x-x+1}{x}}}$

$=\sqrt{\dfrac{x}{\frac{1}{x}}}$

$=\sqrt{x^2}$

$=-x$

(A) $-x$ (B) x (C) 1 (D) $\sqrt{\frac{x}{2}}$ (E) $x\sqrt{-1}$

15. Find the sum of the expression without a calculator:

$$\frac{1}{3+2\sqrt{2}}+\frac{1}{2\sqrt{2}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{5}}+\frac{1}{\sqrt{5}+2}+\frac{1}{2+\sqrt{3}}$$

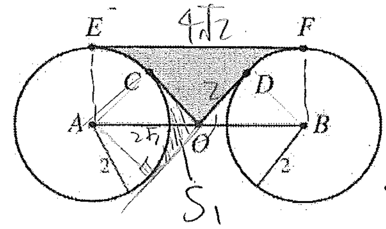
$$=\frac{1}{\sqrt{9}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{3}}$$

$$=\frac{\sqrt{9}-\sqrt{8}}{\sqrt{9}^2-\sqrt{8}^2}+\frac{\sqrt{8}-\sqrt{7}}{\sqrt{8}^2-\sqrt{7}^2}+\frac{\sqrt{7}-\sqrt{6}}{\sqrt{7}^2-\sqrt{6}^2}+\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}^2-\sqrt{5}^2}+\frac{\sqrt{5}-\sqrt{4}}{\sqrt{5}^2-\sqrt{4}^2}+\frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}^2-\sqrt{3}^2}$$

$$=\sqrt{9}-\cancel{\sqrt{8}}+\cancel{\sqrt{8}}-\cancel{\sqrt{7}}+\cancel{\sqrt{7}}-\cancel{\sqrt{6}}+\cancel{\sqrt{6}}-\cancel{\sqrt{5}}+\cancel{\sqrt{5}}-\cancel{\sqrt{4}}+\cancel{\sqrt{4}}-\sqrt{3}$$

$$=3-\sqrt{3}$$

16. Circles centered at A and B each have radius 2, as shown. Point O is the midpoint of $\overline{AB}$, and $OA=2\sqrt{2}$. segments OC and OD are tangent to the circles centered at A and B, respectively, and $\overline{EF}$ is a common tangent. What is the area of the shaded region $ECODF$?



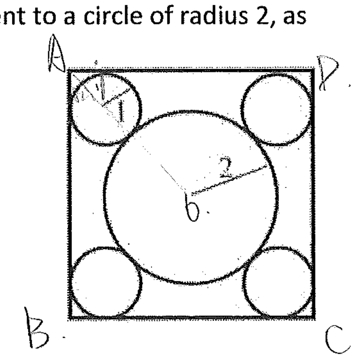
$$S_{ABEF}-\frac{1}{4}S_{\odot A}-\frac{1}{4}S_{\odot B}-S_1$$

$$=4\sqrt{2}\times 2-\frac{1}{2}\cdot 2^2\pi-\left(2^2-\frac{1}{4}\cdot 2^2\pi\right)$$

$$=8\sqrt{2}-2\pi-4+\pi$$

$$=\boxed{8\sqrt{2}-4-\pi}$$

17. Four circles of radius 1 are tangent to two sides of a square and externally tangent to a circle of radius 2, as shown. What is the area of the square?



$$AO=\sqrt{2}+1+2=3+\sqrt{2}$$

$$S_{\square ABCD}=AB^2$$

$$=\left(\frac{3+\sqrt{2}}{\sqrt{2}}\times 2\right)^2$$

$$=\left(3\sqrt{2}+2\right)^2$$

$$=20+12\sqrt{2}$$

Name: Claire Date: 10.29

Math 9 Enriched: Assignment 2.7 Number of Factors and Decimal Representations

1. Indicate the number of factors for each of the following numbers:

a) $N = 824 = 2^3 \cdot 103^1$. $(3+1)(1+1) = 8$. Answer: 8.

b) $N = 4\times25\times36 = 2^4 3^2 5^2$. Answer: 45.

c) $N = 3888 = 2^4 3^5$. 5×6. Answer: 30.

d) $N = 1024 = 2^{10}$. Answer: 11.

e) $N = 2008 = 2^3\times251$. 4×2. Answer: 8.

f) $N = 3^4\times6^3\times8^2 = 2^3\times2^6\times3^4\times3^3 = 2^9\times3^7$. Answer: 80.

g) $N = 2^3(6!) = 6\times5\times2^2\times3\times2\times2^3 = 2^6\times3^5\times5$. $7\times6\times2$. Answer: 84

h) $N = 2^3 3^4 7^4 (125) = 2^3\cdot3^4\cdot5^3\cdot7^4$. $4\times5\times4\times5$. Answer: 400.

i) $N = 4^3\times81^3 = 2^6\times3^{12}$. 7×13. Answer: 91

j) $N = 25! = 2^{22}\cdot3^{10}\cdot5^6\cdot7^3\cdot11\cdot13$ 17.19.23. Answer: 340032

k) $N = 5!\times4!\times3!\times2!\times1 = 5\times4^3\times3^3\times2^4 = 2^8\times3^3\times5$. $9\times4\times2$. Answer: 72

l) $N = (6!)^3\times(10!)^4 = 2^{12}\times3^6\times5^3\times2^{32}\cdot3^{16}\cdot5^8\cdot7^4 = 2^{44}\cdot3^{22}\cdot5^{11}\cdot7^4 = 62100$

2. Given that "N" is a positive integer, find the lowest value of N such that the expression will have the indicated number of factors:

a) $2^3 3^N$ (8 *factors*)
$4\times(N+1) = 8$. $N = 1$.

b) $(8)\times27N$ (48 factors)
$2^3\cdot3^3\cdot N$
$4\times4\times\boxed{3} = 48$. $N = 5^{3-1} = 5^2 = \boxed{25}$

c) $2^3 3^4 N^2$ (56 *factors*)
$56 = 2^3\cdot7$. $7\cdot8$. $2^7 3^6$
$= 2^7\cdot3^6$. $N = 3^2\cdot2^4 = 12$.

3. Given the value for "N", indicate how many of its factors are perfect squares.

a) $N = 2048 = 2^{11}$
6

b) $N = 16\times225\times81 = 2^4\cdot3^6\cdot5^2$
$3\times4\times2 = \boxed{24}$

c) $N = 20! = 2^{18}\cdot3^8\cdot5^4\cdot7^2\cdot11\cdot13\cdot17\cdot19$
$10\times5\times3\times2$ $\boxed{300}$

d) $N = 5^3\times3^7$
5^0 3^0 5^2 3^2 3^4 3^6
2×4
8.

e) $N = 4^3\times6^4\times5^3 = 2^{10}\times3^4\times5^3$
$6\times3\times2 = \boxed{36}$

f) $N = 3^4\times6^3\times18!$
$= 3^7\times2^3\times2^{16}\times3^8\times5^3\cdot7^2\cdot11\cdot13\cdot17$.
$= 2^{19}\cdot3^{15}\times5^3\cdot7^2\cdot11\cdot13\cdot17$
$10\times8\times2\times2 = \boxed{320}$

4. What is the value of $\frac{10^6}{20^3}$? (No Calculators)

$$= \frac{10^6}{2^3\cdot10^3} = \frac{10^3}{2^3} = 5^3 = 125$$

5. What is the greatest prime factor of: $13^{12}+13^{13}+13^{14}+13^{15}$?

$$= 13^{12}(1+13+13^2+13^3)$$
$$= 13^{12}[(1+13)+13^2(1+13)]$$
$$= 13^{12}(1+13)(1+13^2)$$
$$= 13^{12}\cdot14\cdot170.$$
7×2 $2\times5\times17$
$$= 2^2\cdot5\cdot7\cdot13^{12}\cdot\boxed{17}.$$

6. How many of the integers in the interval from 1 to 40 have exactly 4 (Positive) divisors?

2 3 5 7 11 13 17 19. 11

$2^3, 3^3$ 2

$11+2=\boxed{13}$

$2\times2 \to A^1\times B^1$

$1\times4 \to C^3$

7. What is the smallest prime that divides 2047?

$2047 = 23\times89$

$\boxed{23}$

8. How many integers between 100 and 999 divide 96^2?

$96^2 = 2^{10}\cdot3^2$

$\boxed{9}$

2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8, 2^9, 2^{10} ; 3^0, 3^1, 3^2

9

9. If $4!\times4!\times N = 8!$, what is the value of N?

$4\times3\times2\times N = 8\times7\times6\times5$

$N = \boxed{70}$

10. Let $N = 1\cdot3\cdot5\cdot7\cdot9\cdots\cdots163$. What is the largest positive integer "n" such that 3^n is a factor of *N*?

$\left\lfloor\frac{163}{3}\right\rfloor = 54$ $\left\lfloor\frac{163}{3^2}\right\rfloor = 18$ $\left\lfloor\frac{163}{3^3}\right\rfloor = 6$ $\left\lfloor\frac{163}{3^4}\right\rfloor = 2$ $54+13+6+2 = 75$

11. What is the smallest positive integer which is not a factor of 25! ?

prime number > 25 29

12. How many positive integers divide 120 but do not divide 24? Note that for any positive integer "n", 1 divides "n" and "n" divides "n".

$120 = 2^3\cdot3\cdot5 \to 16$

$24 = 2^3\cdot3 \to 4\times2 = 8$

$16-8=\boxed{8}$

13. For how many positive integer values of "n" is *3n - 1* a factor of 2^{2010}?

$3n-1 = 2^1$ $3n-1=2^5$

$n=1$ $n=11$

$3n-1=2^3$ $3n-1=2^7$

$n=3$ $n=43$

$2010\div2 = \boxed{1005}$

14. What is the probability that a randomly selected positive divisor of 2^{100} is a divisor of 2^{50}? Note that for any positive integer "*N*", both 1 and "*N*" are divisors of "*N*". Express your answer as a common fraction.

$2^{100} \to 101$

$2^{50} \to 51$

$\boxed{\frac{51}{101}}$

15. What is the smallest positive integer "*n*" such that 100 divides "*n!*" ?

$\boxed{10}! = 10\times9\times8\times7\times6\times5\times4\times3\times2\times1$

16. An integer "N" is chosen at random from the integers in the interval from 1 to 30, inclusive. What is the probability that "N" is a factor of 30? Express your answer as a common fraction.

$30 = 2\cdot3\cdot5$

$2\times2\times2 = 8$

$\frac{8}{30} = \boxed{\frac{4}{15}}$

17

Recall that $n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$. The maximum value of the integer x such that 3^x divides 30! is:

(A) 30 (B) 14 (C) 13 (D) 10 (E) 4

$\left\lfloor \frac{30}{3} \right\rfloor = 10$ $\left\lfloor \frac{30}{3^2} \right\rfloor = 3$ $\left\lfloor \frac{36}{3^3} \right\rfloor = 1$ $10+3+1=14$

18

Assuming that a, b, c, and d are distinct prime numbers greater than 10, the number of common multiples of $14a^7b^5c^4$ and $98a^3b^{15}d^7$ which are factors of the product of the two numbers is:

(a) 15 (b) 96 (c) 140 (d) 588 (e) 729

$14a^3b^5c$

$2^1 \cdot 7^1 a^3 b^5$

$2 \times 2 \times 4 \times 6 = 96.$

19

Given that $20! = 20 \times 19 \times 18 \times \cdots \times 2 \times 1$ and 2^n is a factor of 20!, then the largest possible value of n is:

(A) 10 (B) 12 (C) 18 (D) 20 (E) 24

$\left\lfloor \frac{20}{2} \right\rfloor = 10$ $\left\lfloor \frac{20}{4} \right\rfloor = 5$ $\left\lfloor \frac{20}{8} \right\rfloor = 2$ $\left\lfloor \frac{20}{16} \right\rfloor = 1$

$10+5+2+1 = 18$

20

The smallest whole number x that has exactly 12 distinct divisors, including 1 and x, can be found in the interval:

$12 = 2^2 \cdot 3^1$

(A) $45 \le x < 55$ (B) $55 \le x < 65$ (C) $65 \le x < 75$ (D) $75 \le x < 85$ (E) $85 \le x \le 90$

$2^3 \cdot 3^2 = 8 \times 9 = 72.$

21

Given that $30! = 30 \times 29 \times 28 \times \cdots \times 2 \times 1$ and 6^n is a factor of 30!, then the largest possible value of n is:

(A) 7 (B) 9 (C) 14 (D) 17 (E) 18

$\left\lfloor \frac{30}{2} \right\rfloor = 15$ $\left\lfloor \frac{30}{4} \right\rfloor = 7$ $\left\lfloor \frac{30}{8} \right\rfloor = 3$ $\left\lfloor \frac{30}{16} \right\rfloor = 1$

$2 = 15+7+3+1 = 26$ $3 = 14$

22

A number N is the product of P distinct prime numbers. The number of positive integer divisors of N is:

(a) $2^{(P-1)}$ (b) $P!$ (c) $2P$ (d) P^2 (e) 2^P.

$N = \underbrace{a^1 \cdot b^1 \cdot c^1 \cdot d^1 \cdots}$

$\Downarrow \; P$

$2^P.$

23 Let $N = 2 \times 4 \times 6 \times 8 \times \cdots \times 48 \times 50$. How many consecutive zeros does the decimal representation of N end with?

$\left\lfloor \frac{50}{10} \right\rfloor = 5$ $\frac{50}{5\ 10}$ $\times 2 = 10$. $5+1=6$.

24 Let $N = 4^8 \times 5^9$. How many digits are there in the decimal representation of N?

$= 2^{16} \times 5^9$.

$= 2^7 \times (2 \times 5)^9$.

$= 128 \times 10^9$.

$3+9 = \boxed{12}$

25 How many digits are there in the decimal representation of 5^{40}?

26 Let N be the result of multiplying 5^{32} by 160^5. How many zeros does the decimal expansion of N end with?

$N = 160^5 \cdot 5^{32}$.

$= 2^{20} \times 10^5 \times 5^{32}$.

$= 5^{12} \times 10^{20} \times 10^5$

$= 5^{12} \times 10^{25}$

$\boxed{15}$

27 Let $N = 2^{17} \times 5^{10}$. What is the total number of digits in the decimal representation of N?

$= 2^7 \times 10^{10}$.

$= 128 \times 10^{10}$ $3+10 = \boxed{13}$

28 How many digits are there in the decimal representation of

$$10 \times 10^2 \times 10^3 \times 10^4 \times \cdots \times 10^{2006} \times 10^{2007}?$$

$1+2+3+\cdots+2007 = \frac{(1+2007)2007}{2} = 2007 \times 1004 = 2015028$

$2015028+1 = \boxed{2015029}$

29 Let x and y be real numbers such that $x^{10} = 11$ and $y^{20} = 100$. What is the value of $x^{20} \times y^{10}$?

$(x^{10})^2 = 11^2$ $(y^{10})^2 = 100$

$x^{20} = 121$ $y^{10} = 10$

$= 121 \times 10$

$= \boxed{1210}$

30 How many digits in total are there in the decimal representation of $(5^5)^5$?

$5^1 = 5$ $5^5 = 3125$ $5^8 = 390625$

$5^2 = 25$ $5^6 = 15625$ $5^9 = 1953125$

$5^3 = 125$ $5^7 = 78125$ $5^{10} = 9765625$

$5^4 = 625$

31 What is the remainder when 8! is divided by 256?

$8! \div 256 = \frac{(8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2)}{256}$ $256 = 2^8$

$= \frac{7 \times 3 \times 5 \times 3}{2}$

$= 157.\boxed{1}$

4 2 1

32 Let a be the number of divisors of $6!$, and let b be the number of divisors of $7!$. What is the value of $\frac{a}{b}$? Express your answer as a common fraction.

	# of factors:
$6! = 2^5 \cdot 3^2 \cdot 5$	$6 \times 3 \times 2 = 36$.
$7! = 2^5 \cdot 3^2 \cdot 5 \cdot 7$	72

$\boxed{\frac{1}{2}}$ //

33 Suppose that we write $(2 \times 2007)^2$ as $a_1 \times a_2 \times \cdots \times a_n$, where $a_1, a_2, \ldots, a_n$ are prime numbers, not necessarily distinct. What is the value of $a_1 + a_2 + \cdots + a_n$? Note that 1 is not a prime.

$(2 \times 2007)^2 = 2^2 \cdot 3^4 \cdot 223^2$

$2+2+3+3+3+3+223+223 = \boxed{462}$ //

34 For any whole number n, the number $n!$ is defined by

$$n! = (1)(2)(3)(4)\cdots(n-1)(n).$$

For example $4! = (1)(2)(3)(4) = 24$.

What is the the remainder when $2! + 3! + 4! + \cdots + 89! + 90!$ is divided by 90?

$90 = 2 \cdot 3^2 \cdot 5 \rightarrow 6!$

$2! + 3! + 4! + 5! = 2 + 6 + 24 + 120 = 152$

$152 \div 90 = 1 \text{ R } \boxed{62}$ //

35

If k is the smallest positive integer such that $(2^k)(5^{300})$ has 303 digits when expanded, then the sum of the digits of the expanded number is

(A) 11 (B) 10 (C) 8 (D) 7 (E) 5

$2^7 = 128$ 3 digits!

$2^{7+300} \cdot 5^{300} = 128 \times 10^{300}$

$1 + 2 + 8 = 11$